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The graded Yang–Baxter relation for the one-dimensional Bariev chain of fermions

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Abstract. The graded version of quantum Yang–Baxter relation for the one-dimensional Bariev chain of fermions is formulated. From this relation the commutativity of the transfer matrices for different values of the spectral parameter follows. Our result is consistent with the applicability of the traditional Bethe ansatz method. Further, the first non-trivial conserved current next to the Hamiltonian is constructed.

Recently, there has been considerable interest in the study of strongly correlated fermion systems. In particular, the discovery of high-T_c superconductivity has greatly stimulated studies of various electron lattice models in one dimension (1D) [1–8], which are exactly soluble using the coordinate Bethe ansatz method [9]. Notably, it has been found that many models, such as the 1D Hubbard model [5], the supersymmetric t–J model [2], and the 1D Bariev chain of fermions [6], exhibit different physical behaviour.

Although these models may be mapped into their equivalent coupled spin chains, it seems to be still interesting to investigate their mathematical structure from the graded version of the quantum inverse scattering method (QISM) [9, 10]. As was emphasized in [11], by studying their relationships with quantum spin chains, one may put on an equal setting the fermion chains and the related coupled spin chains. This has been done by Wadati and his coworkers [11] for the 1D Hubbard model and by Essler and Korepin [8] for the supersymmetric t–J model. However, in contrast to these two well studied models, the algebraic structure of the 1D Bariev chain of fermions remains unexplored so far.

Recently, we have succeeded in constructing the Lax operator as well as the quantum R-matrix for a coupled spin chain which is equivalent to the Bariev chain of fermions [12]. Guided by these results, in this paper we will present the graded version of the quantum Yang–Baxter relation for the Bariev chain of fermions. From this a one-parameter family of commuting transfer matrices is constructed. Our result is consistent with the applicability of the traditional Bethe ansatz method [6]. Further, the first non-trivial conserved current next to the Hamiltonian is constructed.

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To begin with, let us consider a one-dimensional periodic fermion chain of N sites with Hamiltonian [6]

$$H = \sum_{j=1}^{N} [(c_{j\uparrow}^{\dagger} c_{j+1\uparrow} + c_{j+1\uparrow}^{\dagger} c_{j\uparrow}) \exp(\eta n_{j+1\downarrow}) + (c_{j\downarrow}^{\dagger} c_{j+1\downarrow} + c_{j+1\downarrow}^{\dagger} c_{j\downarrow}) \exp(\eta n_{j\uparrow})]. \tag{1}$$

Here $c_{j\alpha}^{\dagger}$ and $c_{j\alpha}$ are, respectively, the creation and annihilation operators of fermions with spin $\alpha(=\uparrow$ or $\downarrow)$ at site j and $n_{j\alpha}$ is the density operator.

Let us first note that the integrability of model (1) is related to the fact that the equations of motion derived from the above Hamiltonian may be cast into the Lax form [13]:

$$L'_j = M_{j+1} L_j - L_j M_j. \tag{2}$$

Here we only write down the L operator

$$L_j(\lambda) = \tilde{L}_j(\lambda) \tilde{\tilde{L}}_j(\lambda) \tag{3}$$

with

$$\tilde{L}_j(\lambda) = \begin{pmatrix} \lambda \exp(\eta) + (\mathrm{i} - \lambda \exp(\eta)) n_{j\uparrow} & 0 & -\mathrm{i}\sqrt{1+\exp(2\eta)\lambda^2} c_{j\uparrow} & 0 \\ 0 & \lambda + (\mathrm{i} - \lambda) n_{j\uparrow} & 0 & \mathrm{i}\sqrt{1+\lambda^2} c_{j\uparrow} \\ \sqrt{1+\exp(2\eta)\lambda^2} c_{j\uparrow}^{\dagger} & 0 & 1 - (1 + \mathrm{i}\lambda \exp(\eta)) n_{j\uparrow} & 0 \\ 0 & -\sqrt{1+\lambda^2} c_{j\uparrow}^{\dagger} & 0 & 1 - (1 + \mathrm{i}\lambda) n_{j\uparrow} \end{pmatrix} \tag{4}$$

and

$$\tilde{\tilde{L}}_j(\lambda) = \begin{pmatrix} \lambda \exp(\eta) + (\mathrm{i} - \lambda \exp(\eta)) n_{j\downarrow} & \sqrt{1+\exp(2\eta)\lambda^2} c_{j\downarrow} & 0 & 0 \\ -\mathrm{i}\sqrt{1+\exp(2\eta)\lambda^2} c_{j\downarrow}^{\dagger} & 1 - (1 + \mathrm{i}\lambda \exp(\eta)) n_{j\downarrow} & 0 & 0 \\ 0 & 0 & \lambda + (\mathrm{i} - \lambda) n_{j\downarrow} & \sqrt{1+\lambda^2} c_{j\downarrow} \\ 0 & 0 & -\mathrm{i}\sqrt{1+\lambda^2} c_{j\downarrow}^{\dagger} & 1 - (1 + \mathrm{i}\lambda) n_{j\downarrow} \end{pmatrix} \tag{5}$$

where λ denotes the spectral parameter. It should be noted that the L operator is a 4×4 supermatrix with parities $P(1) = P(4) = 0$, $P(2) = P(3) = 1$. Thus the quantum Yang–Baxter relation should be understood in the sense of Grassmann algebra [14–16]:

$$R(\lambda, \mu) L_j(\lambda) \otimes_s L_j(\mu) = L_j(\mu) \otimes_s L_j(\lambda) R(\lambda, \mu). \tag{6}$$

Here $\otimes_s$ denotes the Grassmann tensor product defined by

$$(A \otimes_s B)_{ik,jl} = (-1)^{[P(i)+P(j)]P(k)} A_{ij} B_{kl}.$$

After lengthy but straightforward calculation, we find that such an R-matrix does exist:

$$R(\lambda,\mu)=$$

$$\begin{pmatrix}
\rho_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \rho_2 & 0 & 0 & i\rho_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \rho_2 & 0 & 0 & 0 & 0 & 0 & i\rho_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \rho_4 & 0 & 0 & -i\rho_5 & 0 & 0 & i\rho_6 & 0 & 0 & -\rho_9 & 0 & 0 & 0\\
0 & -i\rho_3 & 0 & 0 & \rho_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \rho_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & i\rho_{12} & 0 & 0 & \rho_7 & 0 & 0 & -\rho_{15} & 0 & 0 & -i\rho_5 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_8 & 0 & 0 & 0 & 0 & 0 & -i\rho_{11} & 0 & 0\\
0 & 0 & -i\rho_3 & 0 & 0 & 0 & 0 & 0 & \rho_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -i\rho_{13} & 0 & 0 & -\rho_{15} & 0 & 0 & \rho_{10} & 0 & 0 & i\rho_6 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_8 & 0 & 0 & -i\rho_{11} & 0\\
0 & 0 & 0 & -\rho_{14} & 0 & 0 & i\rho_{12} & 0 & 0 & -i\rho_{13} & 0 & 0 & \rho_4 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & i\rho_{11} & 0 & 0 & 0 & 0 & 0 & \rho_8 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & i\rho_{11} & 0 & 0 & \rho_8 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_1
\end{pmatrix} \tag{7}$$

with

$$\rho_2=\frac{\sqrt{1+h^2\lambda^2}\sqrt{1+h^2\mu^2}}{1+h^2\lambda\mu}\rho_1$$

$$\rho_3=\frac{(\lambda-\mu)h}{1+h^2\lambda\mu}\rho_1$$

$$\rho_4=\frac{\sqrt{1+h^2\lambda^2}\sqrt{1+h^2\mu^2}\sqrt{1+\lambda^2}\sqrt{1+\mu^2}}{(1+\lambda\mu)(1+h^2\lambda\mu)}\rho_1$$

$$\rho_5=\frac{h\sqrt{1+h^2\mu^2}\sqrt{1+\lambda^2}(\lambda-\mu)}{(1+\lambda\mu)(1+h^2\lambda\mu)}\rho_1$$

$$\rho_6=\frac{\sqrt{1+h^2\mu^2}\sqrt{1+\lambda^2}(\lambda-\mu)}{(1+\lambda\mu)(1+h^2\lambda\mu)}\rho_1$$

$$\rho_7=\left(1+\frac{h^2(\lambda-\mu)^2}{(1+\lambda\mu)(1+h^2\lambda\mu)}\right)\rho_1$$

$$\rho_8=\frac{\sqrt{1+\lambda^2}\sqrt{1+\mu^2}}{1+\lambda\mu}\rho_1$$

$$\rho_9=\frac{(\lambda-\mu)(\lambda-h^2\mu)}{(1+\lambda\mu)(1+h^2\lambda\mu)}\rho_1$$

$$\rho_{10}=\left(1+\frac{(\lambda-\mu)^2}{(1+\lambda\mu)(1+h^2\lambda\mu)}\right)\rho_1$$

$$\rho_{11}=\frac{\lambda-\mu}{1+\lambda\mu}\rho_1$$

$$\rho_{12}=\frac{h\sqrt{1+h^2\lambda^2}\sqrt{1+\mu^2}(\lambda-\mu)}{(1+\lambda\mu)(1+h^2\lambda\mu)}\rho_1$$

$$\rho_{13}=\frac{\sqrt{1+h^2\lambda^2}\sqrt{1+\mu^2}(\lambda-\mu)}{(1+\lambda\mu)(1+h^2\lambda\mu)}\rho_1$$

$$\rho_{14} = \frac{(\lambda-\mu)(h^2\lambda-\mu)}{(1+\lambda\mu)(1+h^2\lambda\mu)}\rho_1$$
$$\rho_{15} = \frac{h(\lambda-\mu)^2}{(1+\lambda\mu)(1+h^2\lambda\mu)}\rho_1$$

where $h = \exp\eta$.

We now proceed to establish the relation between the Hamiltonian (1) and the transfer matrix $\tau(\lambda)$, which is the supertrace of the monodromy matrix $T(\lambda)$ defined by

$$T(\lambda) = L_N \ldots L_1. \tag{8}$$

From (6) it follows that

$$R(\lambda,\mu)T(\lambda)\otimes_s T(\mu) = T(\mu)\otimes_s T(\lambda)R(\lambda,\mu). \tag{9}$$

Thus we have

$$[\tau(\lambda), \tau(\mu)] = 0. \tag{10}$$

This implies that one may view $\tau(\lambda)$ as the generating functional of an infinite number of commuting conserved currents, which may be obtained through the expansion of $\ln\tau(\lambda)$ in powers of λ,

$$\ln\tau(\lambda) = \ln\tau(0) + H\lambda + \tfrac{1}{2}(-\mathrm{i})J\lambda^2 + \cdots \tag{11}$$

where J is the first non-trivial conserved current:

$$\begin{aligned}(-\mathrm{i})J = \sum_{j=1}^{N}\{&[(c^\dagger_{j+1\uparrow}c_{j-1\uparrow} - c^\dagger_{j-1\uparrow}c_{j+1\uparrow}) + (\uparrow\to\downarrow)]\\
&+(\exp(\eta)-1)[(c^\dagger_{j+1\uparrow}c_{j-1\uparrow} - c^\dagger_{j-1\uparrow}c_{j+1\uparrow})(n_{j+1\downarrow}+n_{j\downarrow})\\
&+(n_{j\uparrow}+n_{j-1\uparrow})(c^\dagger_{j+1\downarrow}c_{j-1\downarrow} - c^\dagger_{j-1\downarrow}c_{j+1\downarrow})]\\
&+\tfrac{1}{2}(\exp(2\eta)-1)[(c^\dagger_{j\uparrow}c_{j-1\uparrow} + c^\dagger_{j-1\uparrow}c_{j\uparrow})(c^\dagger_{j+1\downarrow}c_{j\downarrow} - c^\dagger_{j\downarrow}c_{j+1\downarrow})\\
&+(c^\dagger_{j\uparrow}c_{j-1\uparrow} - c^\dagger_{j-1\uparrow}c_{j\uparrow})(c^\dagger_{j+1\downarrow}c_{j\downarrow} + c^\dagger_{j\downarrow}c_{j+1\downarrow})\\
&+(c^\dagger_{j+1\uparrow}c_{j\uparrow} + c^\dagger_{j\uparrow}c_{j+1\uparrow})(c^\dagger_{j+1\downarrow}c_{j\downarrow} - c^\dagger_{j\downarrow}c_{j+1\downarrow})\\
&+(c^\dagger_{j+1\uparrow}c_{j\uparrow} - c^\dagger_{j\uparrow}c_{j+1\uparrow})(c^\dagger_{j+1\downarrow}c_{j\downarrow} + c^\dagger_{j\downarrow}c_{j+1\downarrow})]\\
&+(\exp(\eta)-1)^2[(c^\dagger_{j+1\uparrow}c_{j-1\uparrow} - c^\dagger_{j-1\uparrow}c_{j+1\uparrow})n_{j+1\downarrow}n_{j\downarrow}\\
&+n_{j\uparrow}n_{j-1\uparrow}(c^\dagger_{j+1\downarrow}c_{j-1\downarrow} - c^\dagger_{j-1\downarrow}c_{j+1\downarrow})]\}.\end{aligned} \tag{12}$$

In conclusion, we have presented the graded version of the quantum Yang–Baxter relation for the 1D Bariev chain of fermions. This allows us to write down the commuting transfer matrix, which is the generating functional of an infinite number of conserved currents. In particular, the first non-trivial conserved current next to the Hamiltonian is explicitly constructed. Our result is consistent with the applicability of the traditional Bethe ansatz method [6] and may be useful in understanding the completeness of the Bethe eigenvectors. An interesting question which remains open is to derive the Bethe ansatz equations from either the algebraic Bethe ansatz or the analytic Bethe ansatz approach. We hope to return to this question in the near future.

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